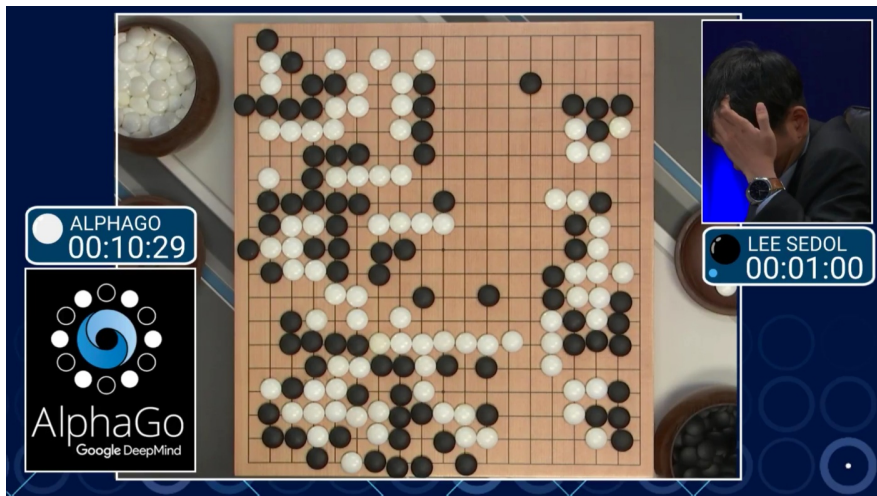
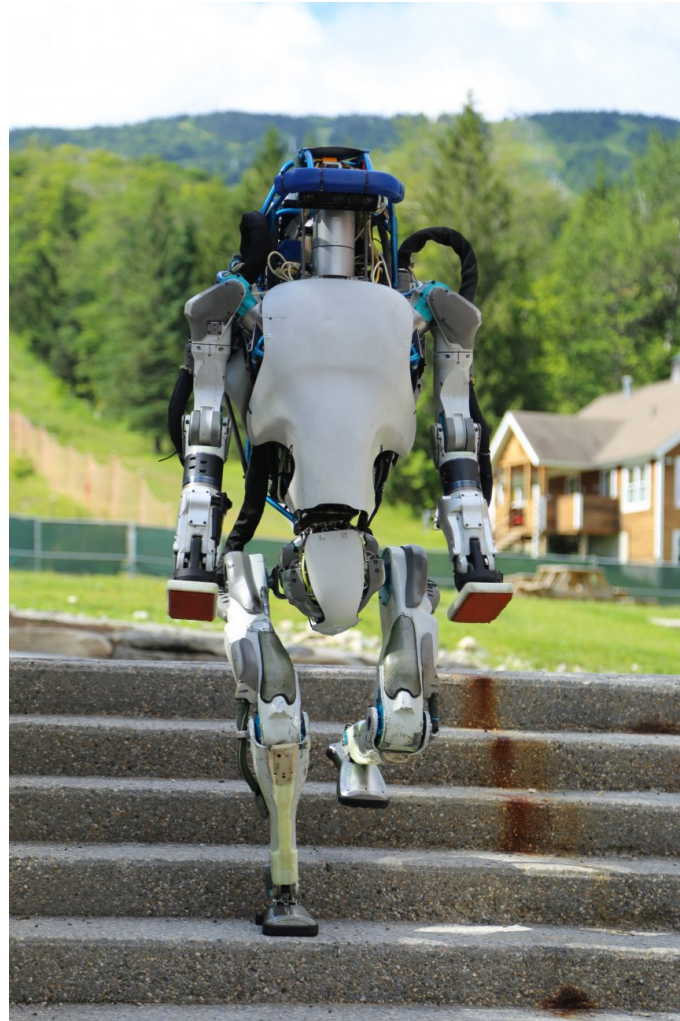
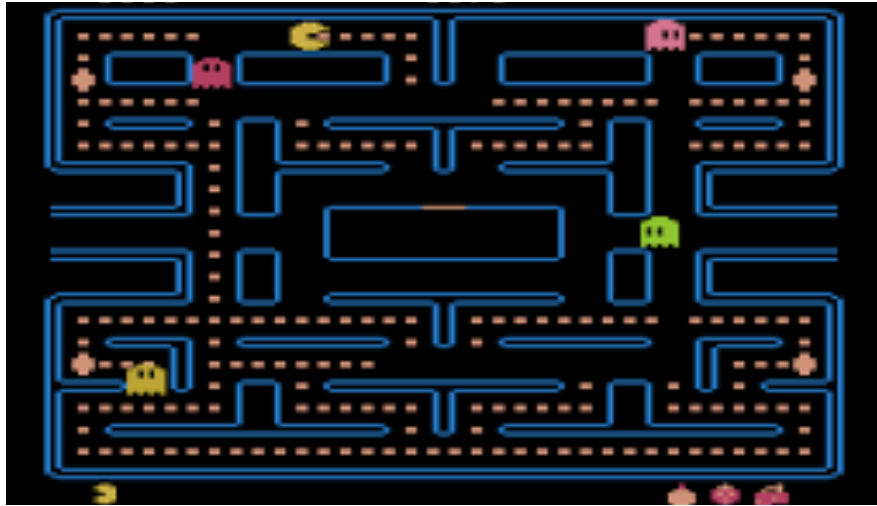


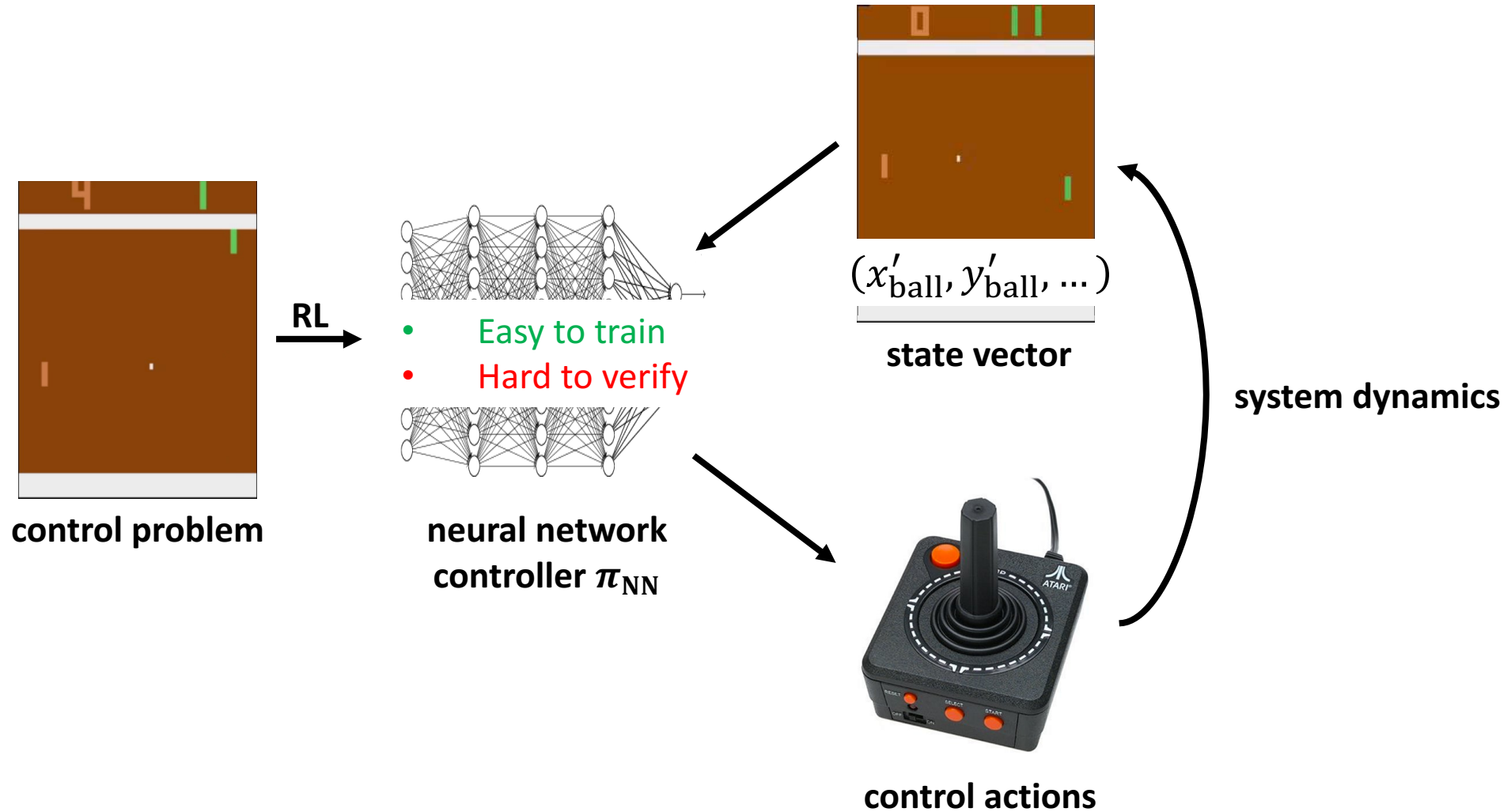
# Verifiable Reinforcement Learning via Policy Extraction

Osbert Bastani, Yewen Pu, Armando Solar-Lezama

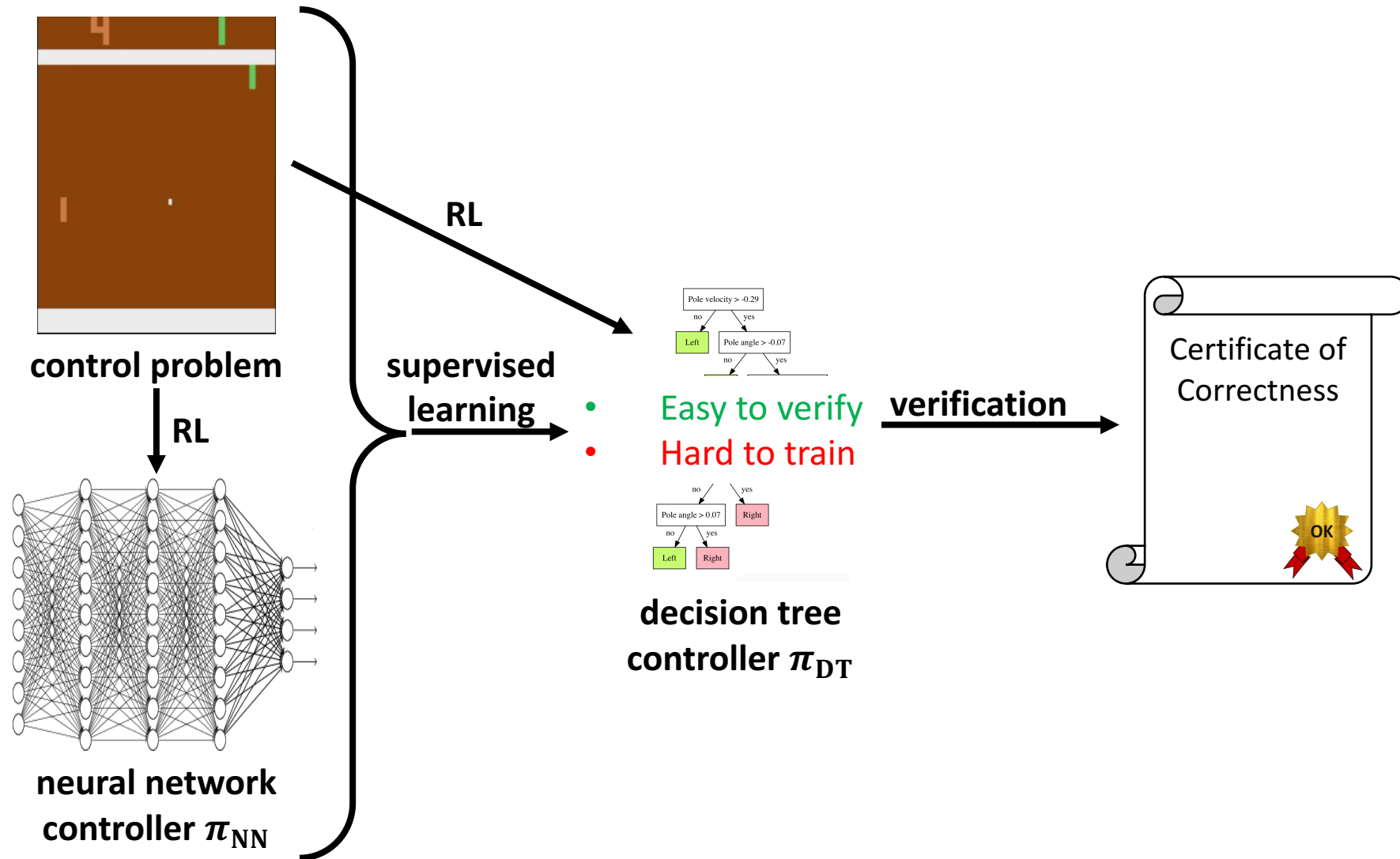
# Deep Reinforcement Learning



# Deep Reinforcement Learning



# Our Approach



Background

# Imitation Learning



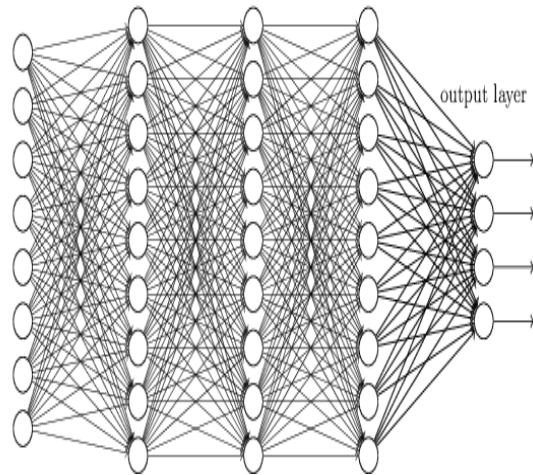
**Demonstrations from Human Expert**



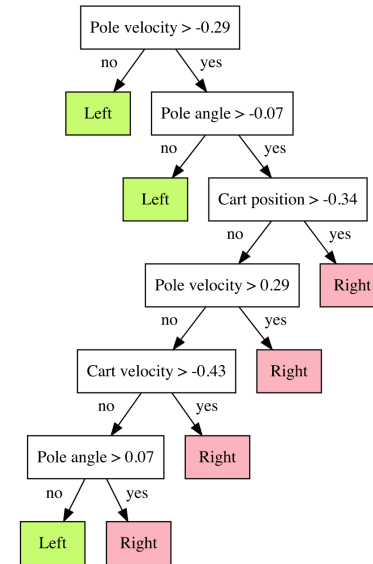
**Controller**

Abbeel & Ng 2004

# Imitation Learning



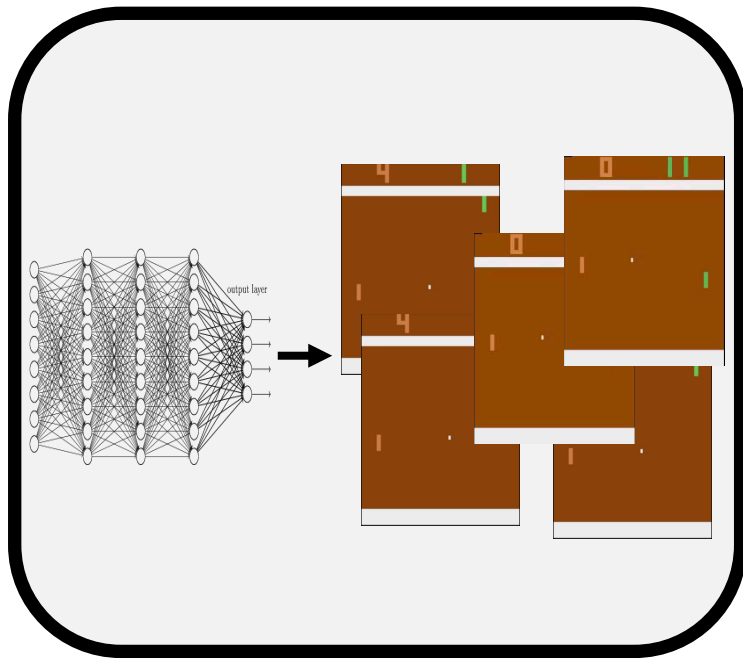
**Demonstrations from Neural Network**



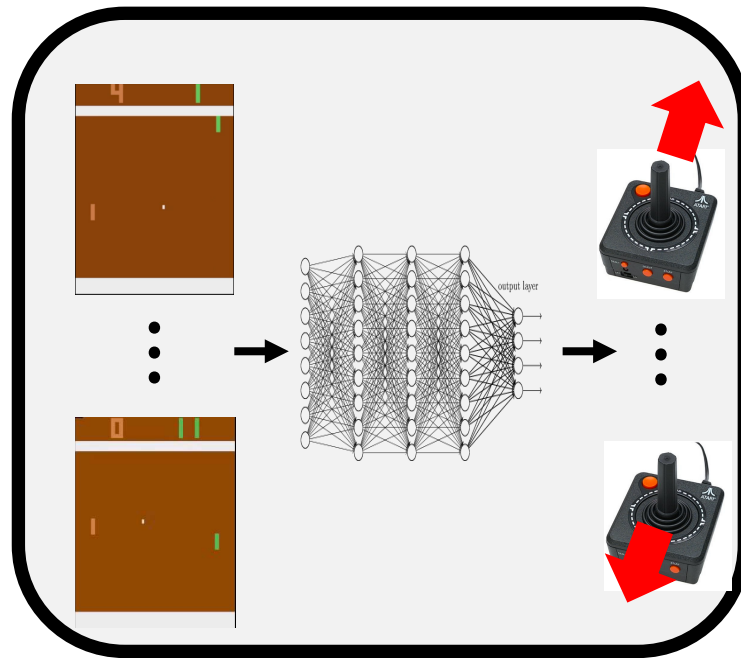
**Decision Tree Controller**

Abbeel & Ng 2004

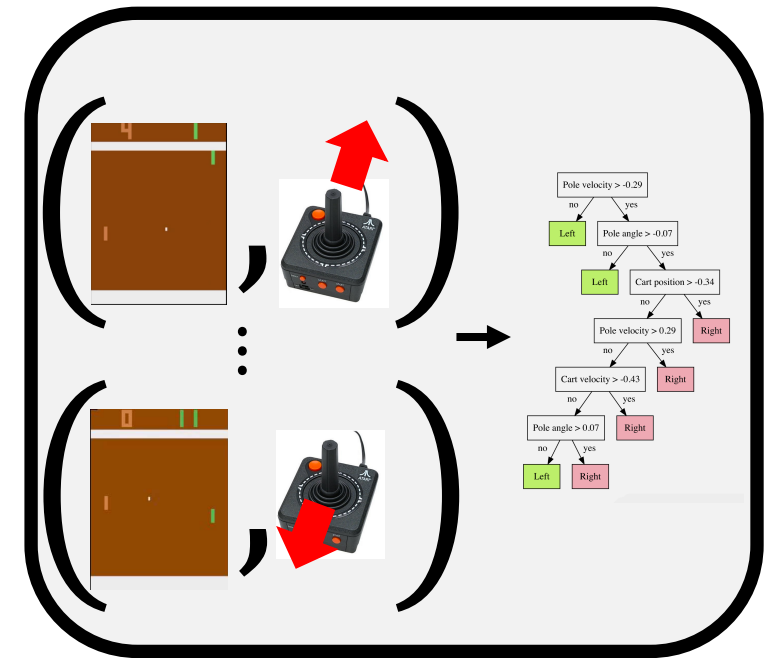
# Imitation Learning



**Step 1:** Use NN to generate states



**Step 2:** Use NN to obtain actions

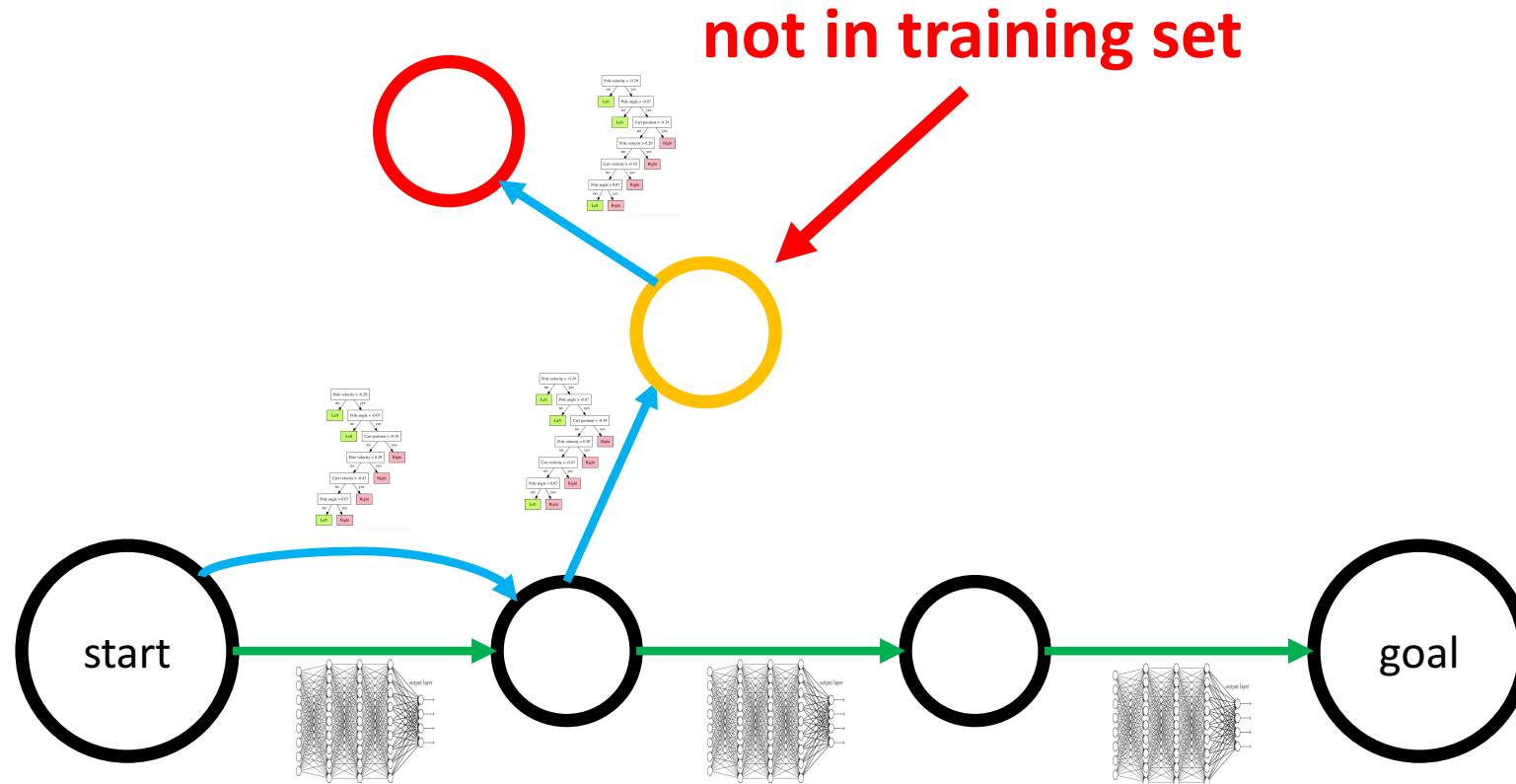


**Step 3:** Use supervised learning to train a decision tree

Ross & Bagnell 2011

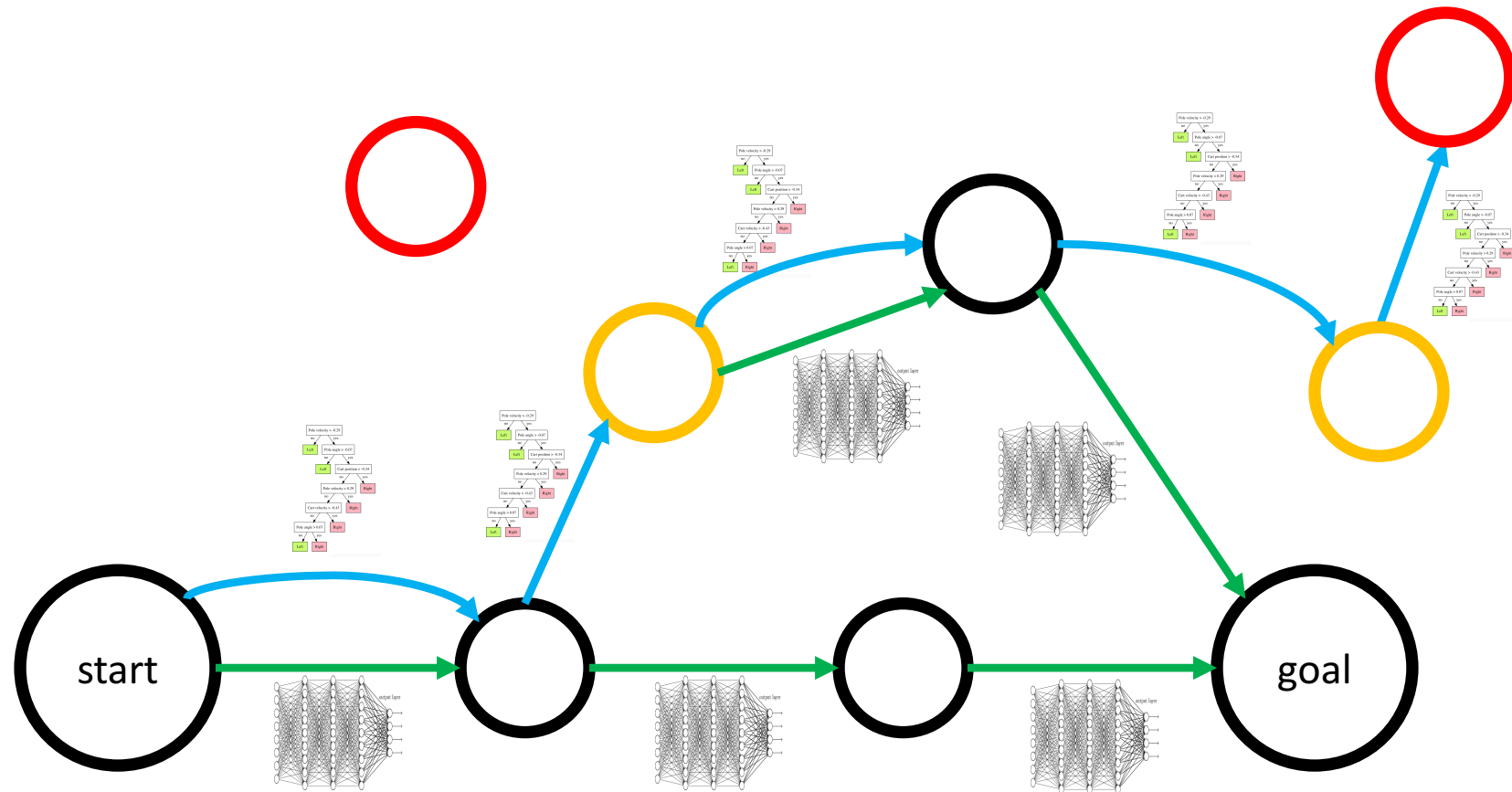


# Imitation Learning



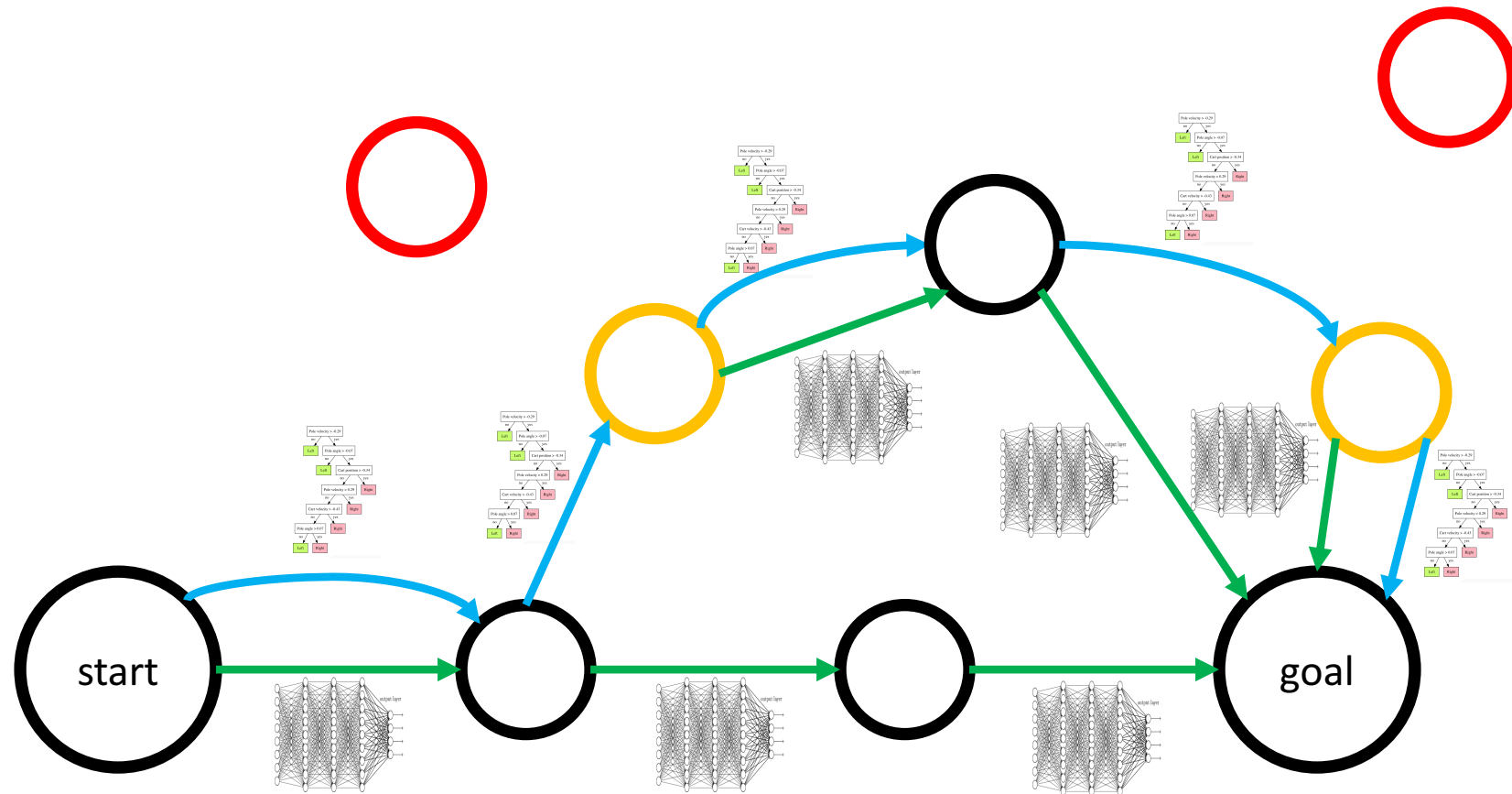
Ross & Bagnell 2011

# Dataset Aggregation (Dagger)



Ross & Bagnell 2011

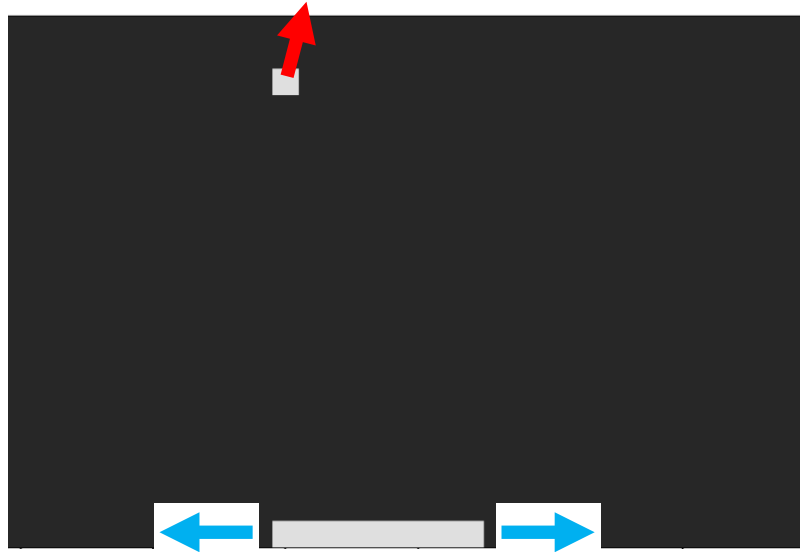
# Dataset Aggregation (Dagger)



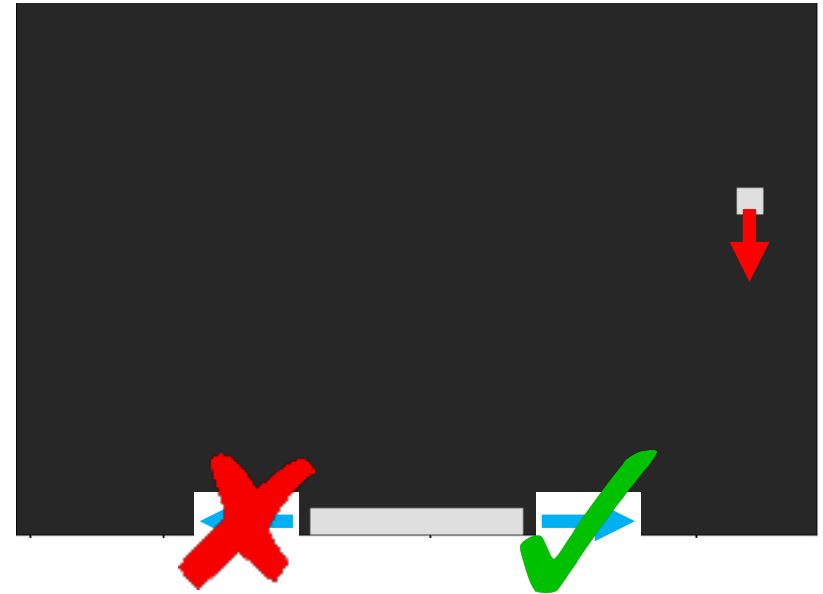
Ross & Bagnell 2011

# Viper Algorithm

# Insight: Critical States



**actions are similar  
(non-critical state)**

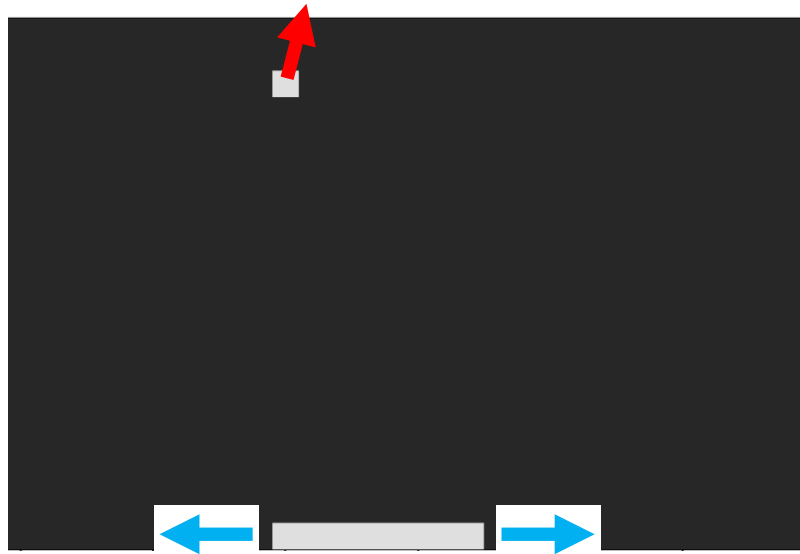


**must move right!  
(critical state)**

# Our Approach: Leverage the $Q$ -Function

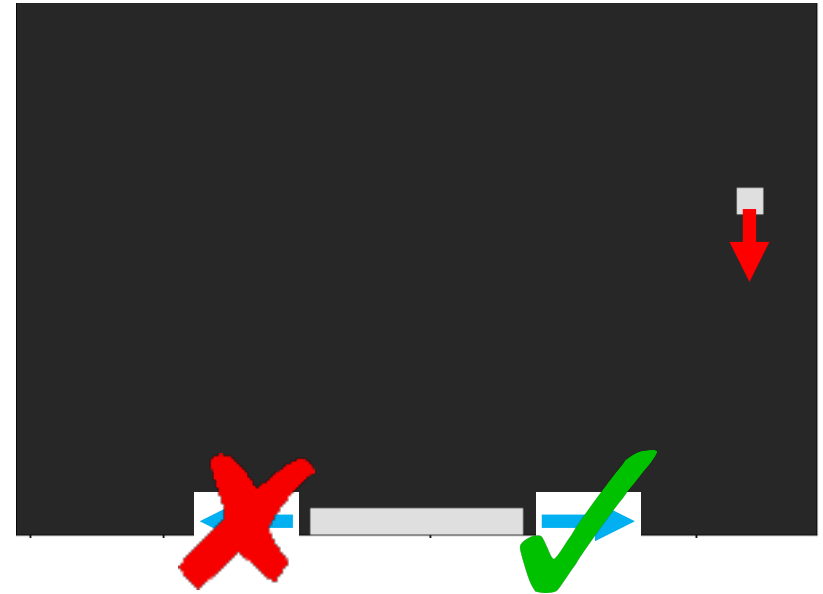
$Q(s, a)$  = “how good is action  $a$  in state  $s$ ?”  $\in \mathbb{R}$

# Our Approach: Leverage the $Q$ -Function



**non-critical state (low priority)**

$$\underbrace{Q(s, \pi_{\text{NN}}(s))}_{\text{optimal } Q \text{ value}} \approx \underbrace{\min_{a \in A} Q(s, a)}_{\text{worst-case } Q \text{ value}}$$



**critical state (high priority)**

$$\underbrace{Q(s, \pi_{\text{NN}}(s))}_{\text{optimal } Q \text{ value}} \gg \underbrace{\min_{a \in A} Q(s, a)}_{\text{worst-case } Q \text{ value}}$$

# Viper Algorithm

- DAgger treats all state-action pairs equally:

$$\pi_{\text{DT}} = \arg \min_{\pi} \sum_{s \in D} \mathbb{I}[\pi(s) = \pi_{\text{NN}}(s)]$$

- Viper weights state-action pairs by the  $Q$ -function:

$$\pi_{\text{DT}} = \arg \min_{\pi} \sum_{s \in D} \left( \underbrace{Q(s, \pi_{\text{NN}}(s))}_{\text{optimal } Q \text{ value}} - \underbrace{\min_{a' \in A} Q(s, a')}_{\text{worst-case } Q \text{ value}} \right) \mathbb{I}[\pi(s) = \pi_{\text{NN}}(s)]$$



# Theoretical Guarantees

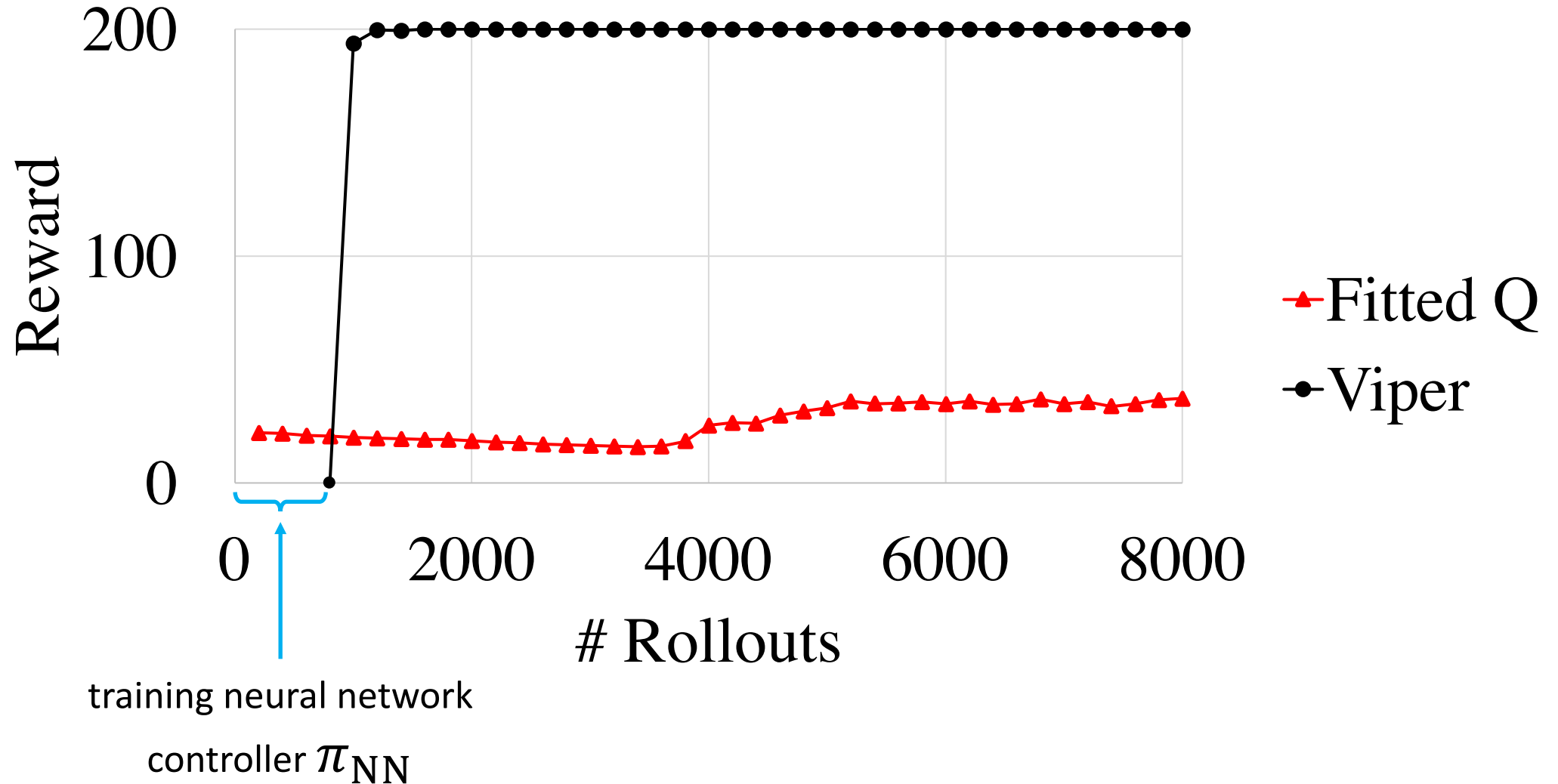
**Theorem.** For any  $\delta > 0$ , there exists a policy  $\hat{\pi} \in \{\hat{\pi}_1, \dots, \hat{\pi}_N\}$  such that

$$J(\hat{\pi}) \leq J(\pi^*) + T\epsilon_N + \tilde{O}(1)$$

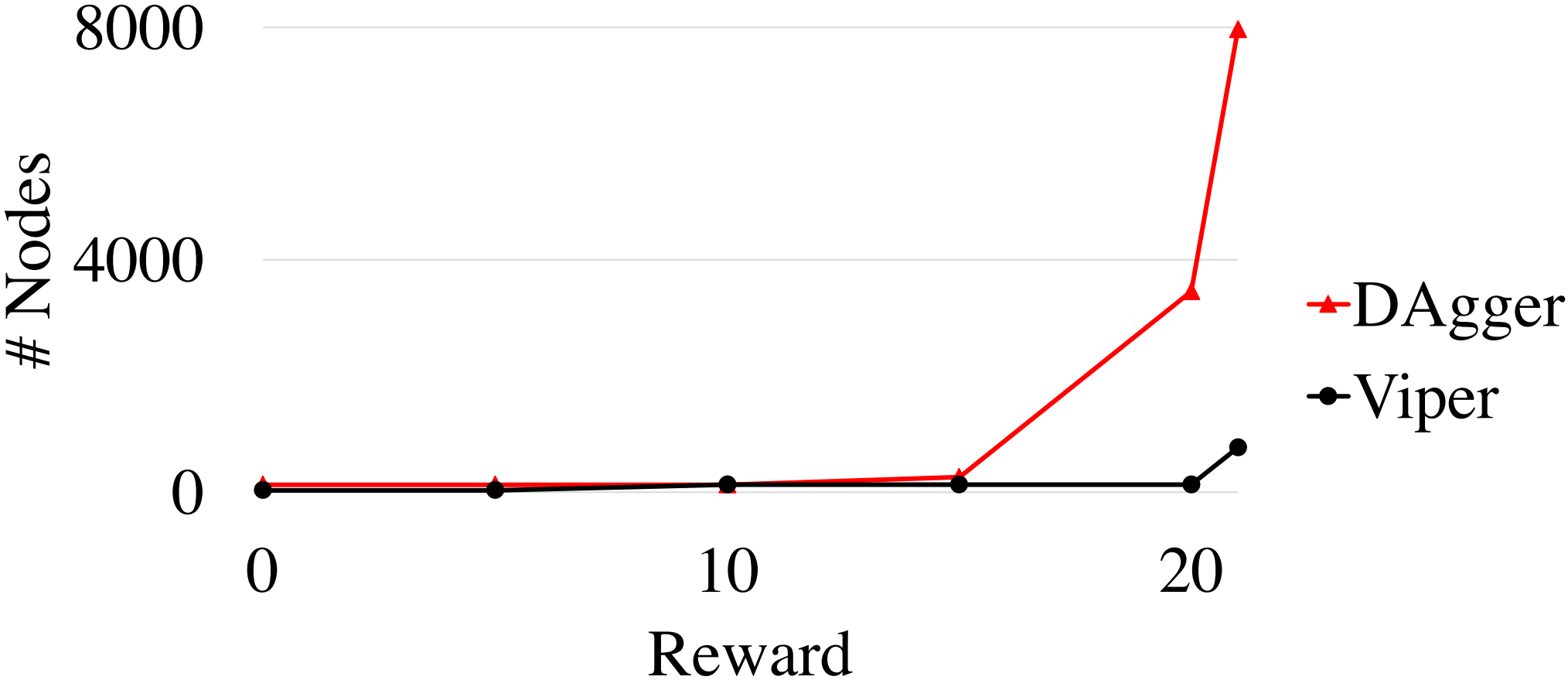
with probability at least  $1 - \delta$ , as long as  $N = \tilde{\Theta}(\ell_{max}^2 T^2 \log(1/\delta))$ .

Evaluation

# vs. Decision Trees via RL (on Cart-Pole)

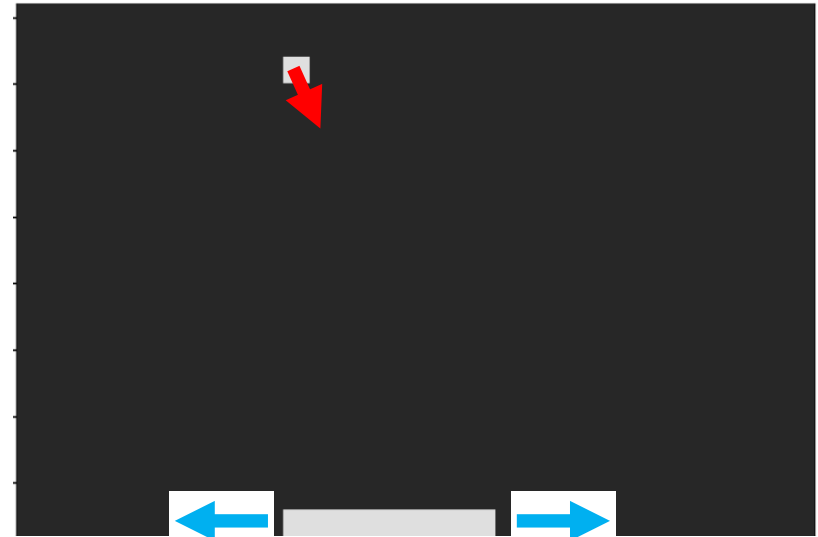


# vs. to DAgger (on Atari Pong)



# Verifying Correctness of a Toy Pong Controller

- **Toy Pong**
  - states =  $\mathbb{R}^5$
  - actions = {left, right, stay}
- **Neural network:**
  - trained using policy gradients
  - 600 neurons
- **Decision tree:**
  - extracted using Viper
  - 31 nodes



# Verifying Correctness of a Toy Pong Controller

- **Inductive invariant:**

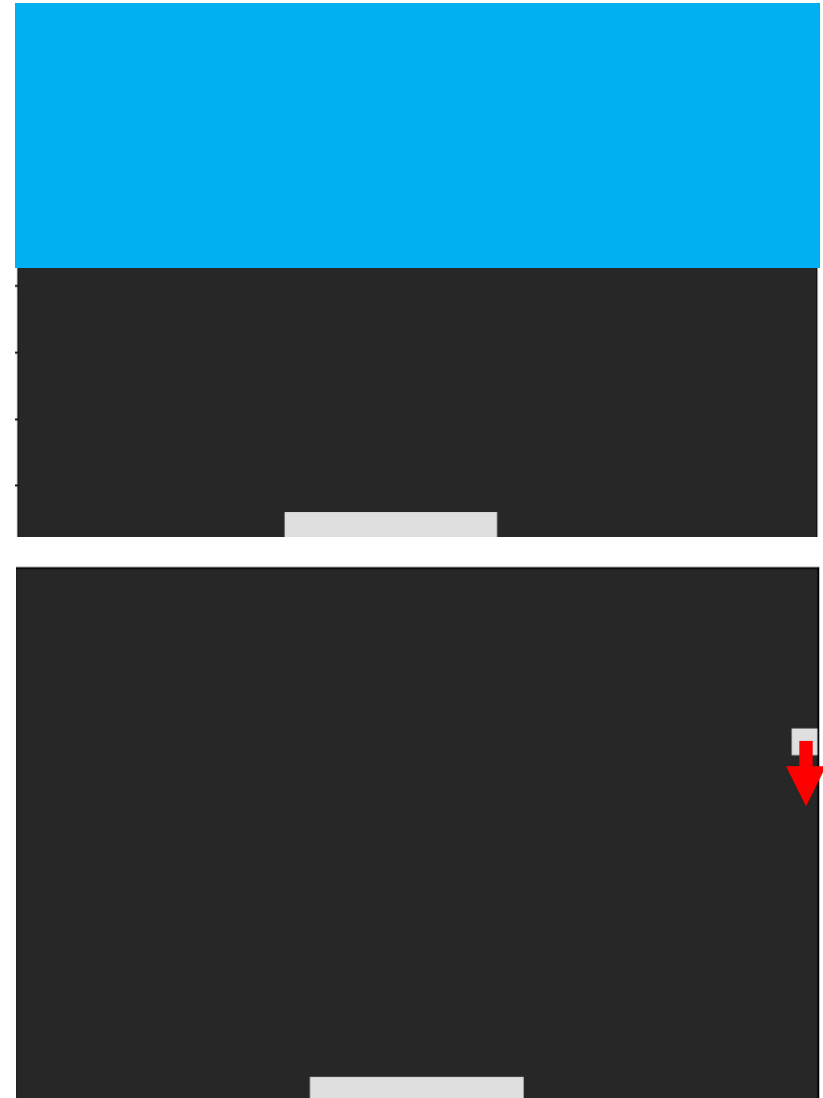
$$s(0) \in \text{blue} \Rightarrow s(t) \in \text{blue}$$

- **Verification algorithm**

- dynamics are piecewise linear
- SMT formula over linear arithmetic
- solved by Z3 in < 5 seconds

- **Results:**

- error when ball starts on the right
- fixed when paddle is slightly longer!



# Conclusion

Verifiability is critical to enabling application of deep reinforcement learning to safe-critical systems